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On the movement of a spherical particle in vertically oscillating liquid

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The movement of a heavy spherical particle in vertically oscillating liquid is studied by means of an analog computer. Using the Oseen–Tchen–Houghton force-balance model, it is shown that in the case of asymmetrical oscillations the particle may be made quasi-stationary with a finite amplitude of oscillation. It is suggested that under such conditions a mean movement of the particle in the upward direction is also possible.

It is known that the mean relative velocity of settling (or rising) of particles in a liquid medium is retarded if the liquid is made to execute sinusoidal vertical oscillations. This effect has been widely studied theoretically (Rozen 1958; Houghton 1963, 1966, 1968; Molerus 1964; Baird, Senior & Thompson 1967; Molerus & Werther 1968; Boyadzhiev & Sapoundjiev 1969) and also confirmed experimentally (Baird, Senior & Thompson 1967; Brush, Ho & Singamesetti 1963; Trunstall & Houghton 1968). By simulating the particle movement on an analog computer it was shown by Boyadzhiev & Sapoundjiev (1969) that the maximal retardation effect which may be achieved upon sinusoidal vertical oscillation of the medium with frequency ω and amplitude A is quasi-stationarity, that is, 'levitation' of the particle with respect to the medium. This could be realized only in the extreme case when $\omega \rightarrow \infty$ at $A \neq 0$.

Let us assume now that the vertical oscillations of the fluid medium are not sinusoidal in character, as in all studies hitherto published, but correspond to an asymmetrical periodic movement with a mean upward velocity greater than the mean downward one:

$$|v_m|_+ > |v_m|_-.$$

The trajectory of an arbitrary element of the medium undergoing such a periodic movement at a sufficient distance from the particle is shown on figure 1, curve 2. The liquid medium is moving faster in the positive direction (upwards) than in the negative one (downwards). For comparison the trajectory of the same element moving sinusoidally is also shown on figure 1, as curve 1.

Taking into account the fact that the retardation effect is caused by the variation of the velocity of the particle relative to the medium it may be assumed on the basis of general considerations that the above-mentioned non-sinusoidal pulsations would bring the particle to quasi-stationarity at a finite value of the oscillation amplitude. In other words, for such types of oscillation, particle

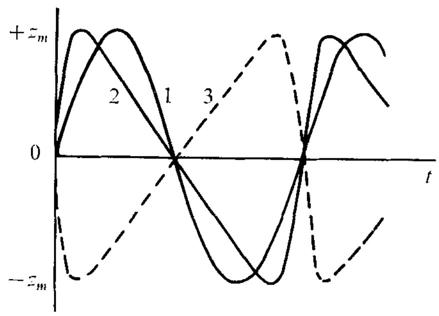

FIGURE 1

levitation will be achieved at a certain oscillation amplitude, and above this it will move in the direction opposite to that of gravity or will be kept quasi-stationary in the bulk. Under these conditions, the relative particle velocity will be higher during the fast upward movement of the medium and hence the drag resistance given by a nonlinear law will be larger than when the medium moves downwards. For such 'asymmetrical behaviour' of the particle during the whole pulse period it may be assumed that above a certain oscillation amplitude the absolute downward displacement of a heavy particle will be smaller than its displacement in opposite direction, regardless of the gravitational force. It may also be assumed that in the opposite case, i.e. for $|v_m|_+ < |v_m|_-$ (see figure 1, curve 3), the retardation effect will decrease and above certain oscillation amplitude a downward mean velocity greater than that due to gravity alone will be observed.

In order to prove this hypothesis, an analog model similar to that discussed earlier (Boyadzhiev & Sapoundjiev 1969) in connexion with computer solution of the same equation applied to a particle moving in a sinusoidally oscillating medium was used. This equation, simulated and investigated by means of a MEDA 40 analog computer, was the following:

$$\frac{dv_p}{dt} = \frac{1{\cdot}5}{\delta + 0{\cdot}5}\frac{dv_m}{dt} - \frac{1-\delta}{\delta + 0{\cdot}5}g - \frac{3C_w}{4d_p(\delta + 0{\cdot}5)}(v_p - v_m)^2 \operatorname{sgn}(v_p - v_m), \tag{1}$$

where δ is the density ratio ρ_p/ρ_m, d_p is the particle diameter, g is the acceleration due to gravity and the subscripts p and m refer to the particle and fluid medium respectively. The drag coefficient C_w was determined from the particle Reynolds number $Re = |v_p - v_m| \, d_p \rho_m / \mu_m$ according to the scheme

$$C_w = \left\{ \begin{array}{lll} 24/Re & \text{for} & Re < 1, \\ f(Re) & \text{for} & 1 < Re < 10^3, \\ 0{\cdot}5 & \text{for} & Re > 10^3, \end{array} \right\} \tag{2}$$

in which the empirical function $f(Re)$ was taken from Vignes (1965) and simulated by means of a functional generator. The function $v_m = \phi(t)$ describing the velocity of a medium oscillating asymmetrically was simulated in a similar manner by a functional generator.

A large number of solutions of (1) were obtained for solid spheres of various

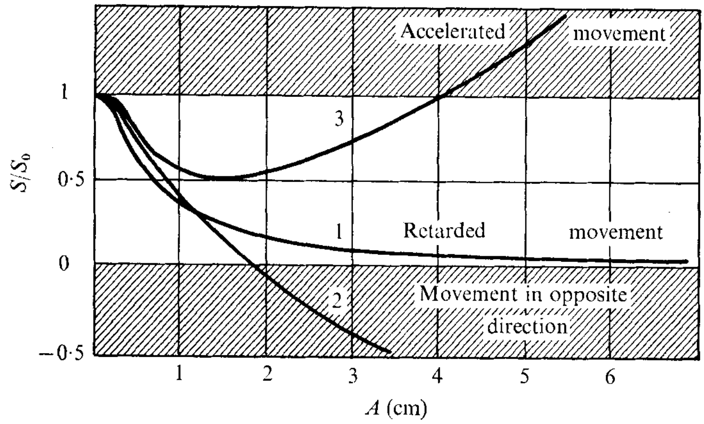

FIGURE 2

sizes and densities. A typical example of particle behaviour for a steel bead ($\rho_p = 8\,\mathrm{Mg/m^3}, d_p = 0{\cdot}2\,\mathrm{cm}$) in oscillating water ($\rho_m = 1\,\mathrm{Mg/m^3}, \mu_m = 0{\cdot}01\,\mathrm{N/m\,s}$) is shown in figure 2. The oscillating frequency was $25\,\mathrm{s^{-1}}$ and the types of oscillations those corresponding to the curves 1, 2 and 3 on figure 1. During every pulse period, when $A \approx 2{\cdot}5\,\mathrm{cm}$, the relative particle velocity achieved values as high as 150 cm/s in both directions and hence the particle Reynolds number ranged between 0 and 3000. For comparison, in motionless water the final settling velocity is 61 cm/s.

The solutions obtained for each case are presented on figure 2 as the change in the retardation effect S/S_0 versus the oscillation amplitude A. The retardation effect is defined as the ratio of the mean distance S travelled by the particle in the oscillating liquid to the distance S_0 which would be covered in a non-oscillating medium.

The results obtained confirmed the hypothesis that quasi-stationarity of the bead is possible (at $A = 1{\cdot}9\,\mathrm{cm}$ for the example shown), and that at higher oscillation amplitudes the bead will move in the reverse direction, i.e. upwards. This means that for $A > 1{\cdot}9$ the steel bead will rise upwards in the vertically oscillating water. In the opposite case, with $|v_m|_+ < |v_m|_-$, the retardation effect decreases when the oscillation amplitude increases, and above a certain oscillation amplitude (in the example for $A > 4{\cdot}2\,\mathrm{cm/s}$) an acceleration effect is observed, i.e. the steel bead will settle faster in an oscillating medium than in a steady one (see figure 2, curve 3). For comparison the corresponding particle behaviour for sinusoidal oscillations of the medium is also shown on figure 2 (curve 1).

Taking into account the fact that the accuracy of the computer solution is quite satisfactory, the validity of the solutions obtained will be determined solely by the adequacy with which the model considered represents the real process. Equation (1), suggested by Houghton, is in fact a simplified modification of the general Oseen–Tchen equation. The differences between the latter and equation (1), and the simplifications introduced have been discussed in detail elsewhere (Houghton 1963; Boyadzhiev & Sapoundjiev 1969).

In order to achieve the simplification in question, some parameters in the Oseen–Tchen equation have been replaced by numerical values obtained in experiments with particles in steady motion relative to the medium. Thus, for example, the coefficient of virtual mass (χ) is replaced by 0·5 in equation (1), and the function $C_w = f(Re)$ is assumed to be identical to that for particles in steady motion. Also, in equation (1) the so-called Basset term has been neglected. All modifications are due to the lack of data valid for these transient conditions. Taking into account the 'high degree of non-stationarity' of the phenomenon discussed, it is evident that such an assumption is rather arbitrary, and hence the models, solutions and conclusions derived on the basis of these assumptions must be considered with great caution. It should be added that such an approach to the phenomenon in question is less accurate than the full description of the hydrodynamics of the particle–medium system. Unfortunately, however, the present state of the turbulence theory renders the possibility of using the correct approach rather remote.

Despite these shortcomings, it may be expected that the assumptions and simplifications mentioned do not cause an essential alteration of the model under consideration. Such a statement is supported by the experimental studies of Trunstall & Houghton (1968) on the retardation effect for media oscillating sinusoidally. These authors have shown that for small beads the differences between the experimental values and those calculated on the basis of the model are insignificant, whereas for large spheres the experimental retardation effect is larger than the theoretically expected one. As in both cases (sinusoidal and asymmetric oscillations) the solution is based on the same model described by the simplified equation (1), the assumption for possible existence of such an 'anti-gravitational hydrodynamic effect' or at least a stable quasi-stationarity is justified. Definitive conclusions may be drawn only after a rigorous experimental study of this problem.

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